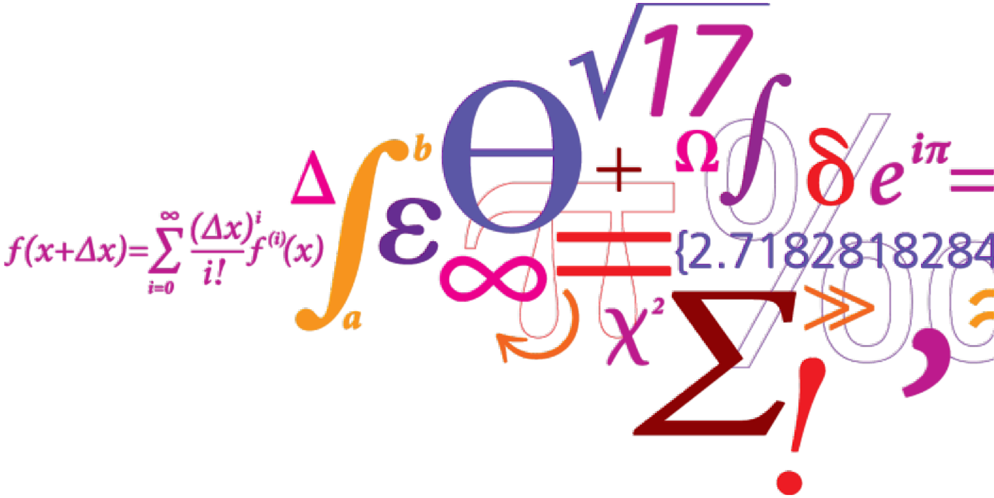
Unveiling Supply Curves in Electricity Markets:
A Bayesian Inference Approach

Lesia Mitridati, Pierre Pinson

EPSRC HubNet Risk Day,
March 6th, 2018

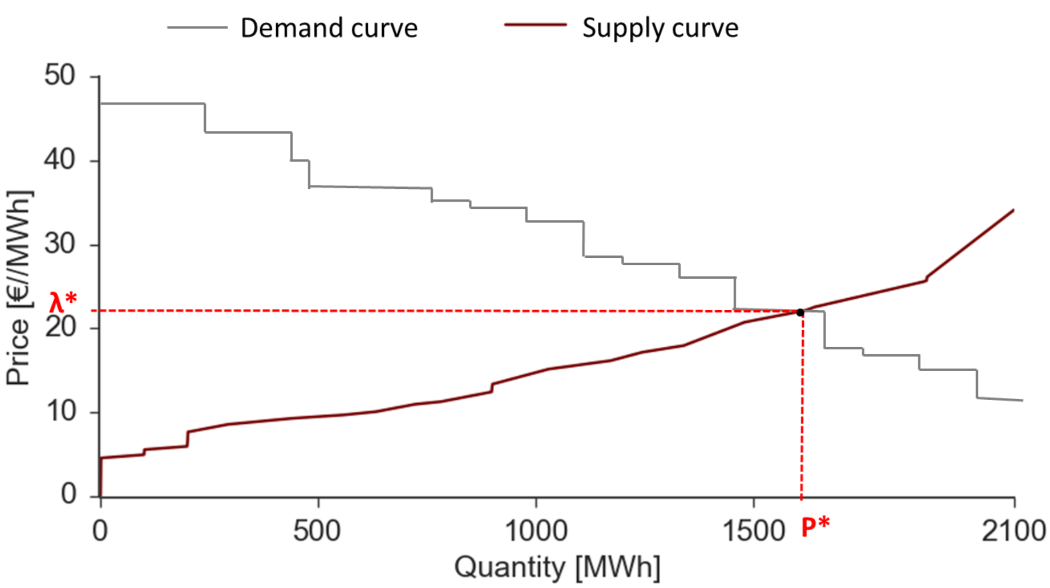


(with acknowledgement to all funding sources and collaborators)

DTU Electrical Engineering
Department of Electrical Engineering

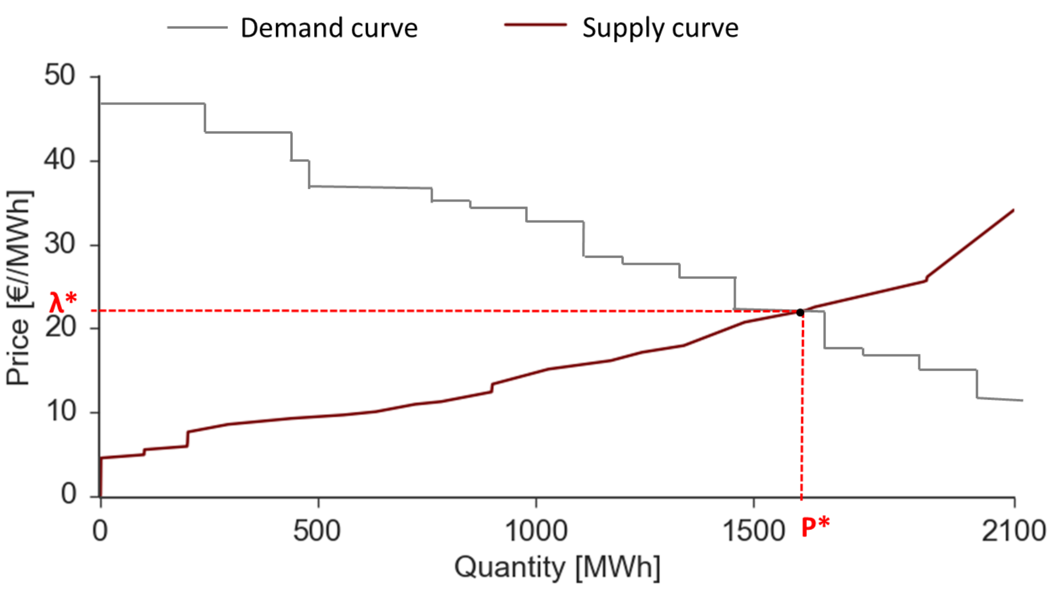
Perfect Competition in Electricity Markets

- **Perfect competition** in the day-ahead market
- All producers offer their true marginal cost



Strategic Offering in Electricity Markets

- A large producer participating in the day-ahead market
- Can exercise **”market-power”** - modify market equilibrium to increase their profit



Strategic Offering in Electricity Markets: Price Maker

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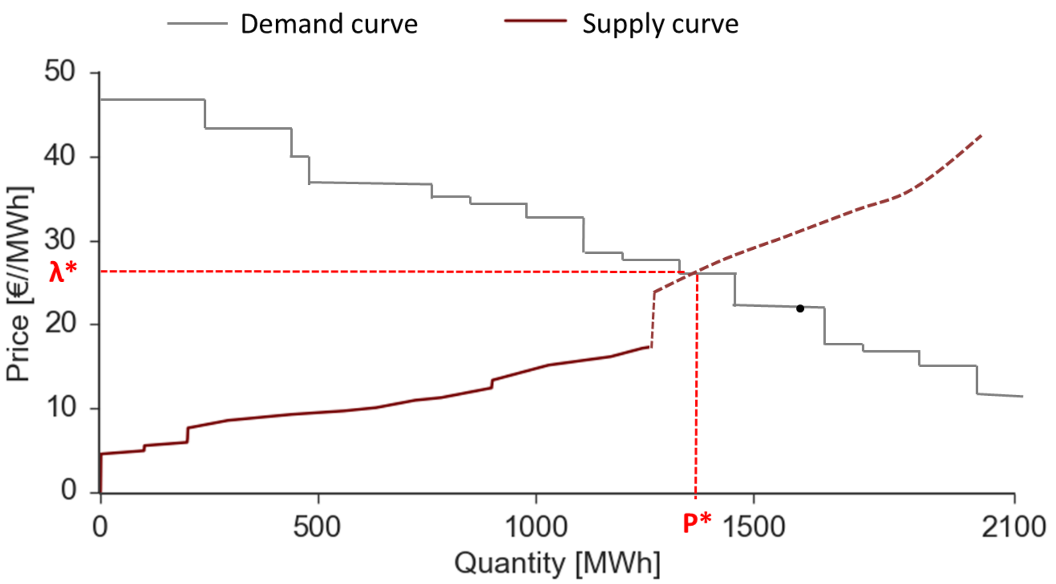

Strategic Offering in Electricity Markets: Data Available?

- Model the behaviour of rival participants: **Aggregate supply curve**

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  - Market data observations

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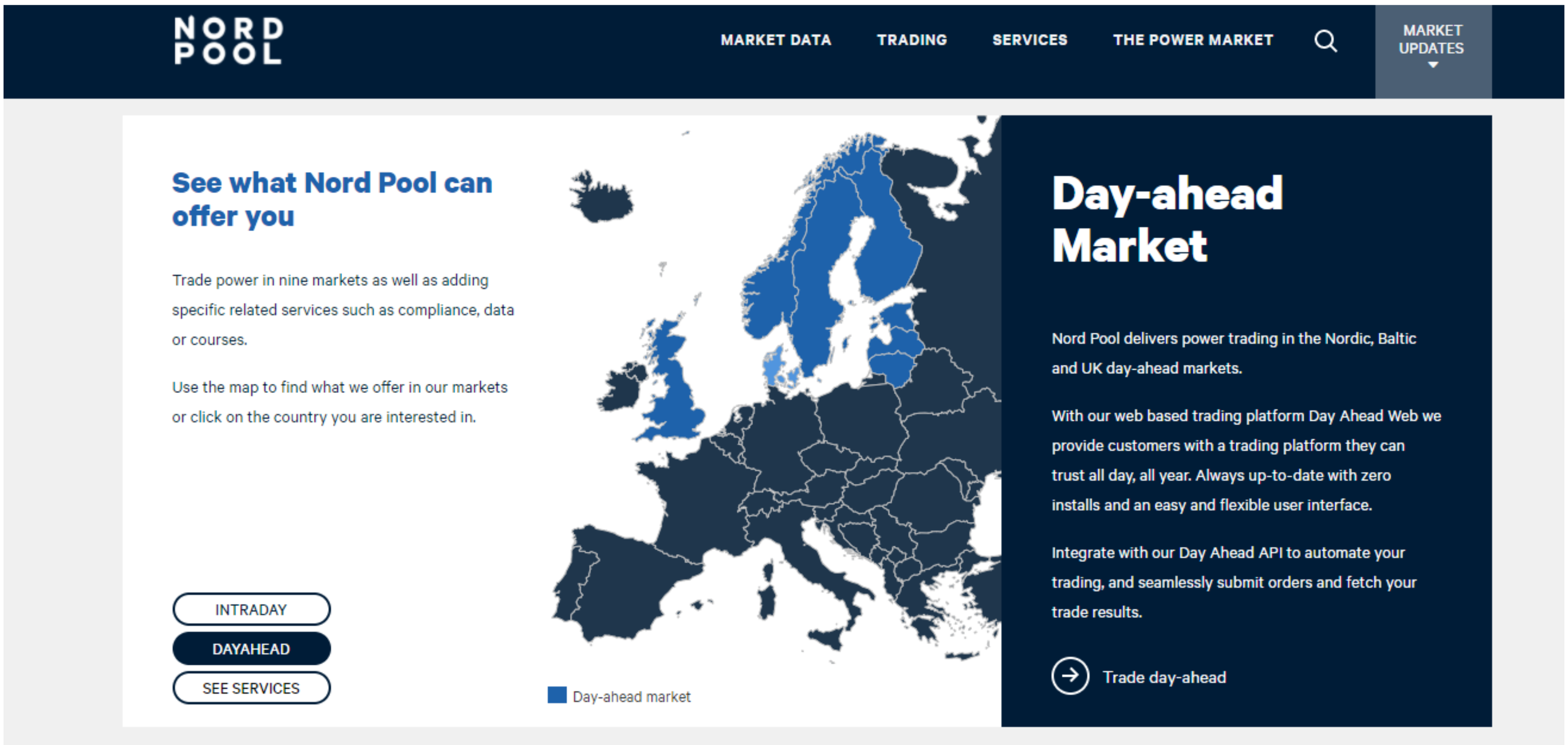
Market Data



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SERVICES
THE POWER MARKET
MARKET UPDATES
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Trade power in nine markets as well as adding specific related services such as compliance, data or courses.
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SEE SERVICES
Day-ahead market
Day-ahead Market
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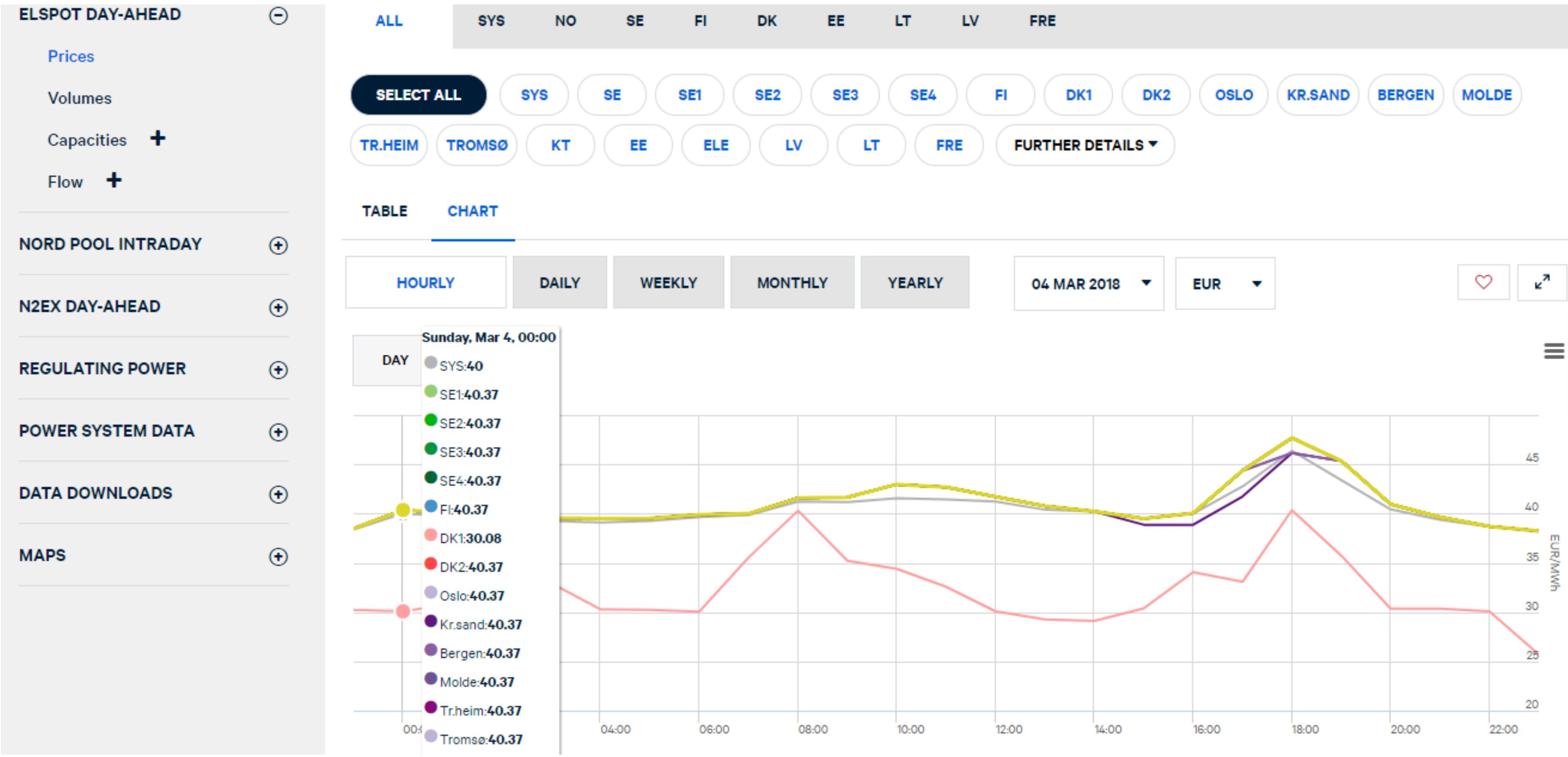
Integrate with our Day Ahead API to automate your trading, and seamlessly submit orders and fetch your trade results.
Trade day-ahead

Market Data: Prices



Market Data: Volumes

DTU

ELSPOT DAY-AHEAD

- Prices
- Volumes
- Capacities +
- Flow +

NORD POOL INTRADAY

N2EX DAY-AHEAD

REGULATING POWER

POWER SYSTEM DATA

DATA DOWNLOADS

MAPS

ALL NO SE FI DK EE LT LV FRE

SELECT ALL | TURNOVER AT SYSTEM PRICE | NO1 BUY | NO1 SELL | NO2 BUY | NO2 SELL | NO3 BUY | NO3 SELL | NO4 BUY | NO4 SELL | NO5 BUY | NO5 SELL | SE1 BUY | SE1 SELL | SE2 BUY | SE2 SELL | SE3 BUY | SE3 SELL | SE4 BUY | SE4 SELL | FI BUY | FI SELL | FRE BUY | FRE SELL | DK1 BUY | DK1 SELL | DK2 BUY | DK2 SELL | EE BUY | EE SELL | LV BUY | LV SELL | LT BUY | LT SELL

TABLE

HOURLY | DAILY | WEEKLY | MONTHLY | YEARLY | 04 MAR 2018

MWh

All hours are in CET/CEST. Last update: **Yesterday 12:42 CET/CE**

04-03-2018	Turnover at system price	NO1 Buy	NO1 Sell	NO2 Buy	NO2 Sell	NO3 Buy	NO3 Sell	NO4 Buy	NO4 Sell	NO5 Buy	NO5 Sell	SE1 Buy
00 - 01	49 596,8	6 183,7	2 047,2	5 701,0	7 615,8	3 496,4	1 932,7	2 205,2	3 180,6	2 354,7	4 867,2	1 396,4
01 - 02	48 973,4	6 093,1	2 038,4	5 657,3	7 453,9	3 473,5	1 867,9	2 179,8	3 175,6	2 330,2	4 744,9	1 386,2
02 - 03	48 612,2	6 029,5	2 024,6	5 606,4	7 391,9	3 449,5	1 747,8	2 164,5	3 168,7	2 289,2	4 549,2	1 383,2
03 - 04	48 562,9	6 007,7	2 022,7	5 582,3	7 374,8	3 452,1	1 733,0	2 161,5	3 171,3	2 282,0	4 476,7	1 385,3
04 - 05	48 575,1	5 992,1	2 010,9	5 422,1	7 363,1	3 449,2	1 724,2	2 165,7	3 172,9	2 280,1	4 442,4	1 398,0
05 - 06	48 793,2	6 020,7	2 024,8	5 060,8	7 368,3	3 471,5	1 764,9	2 177,7	3 172,6	2 304,8	4 524,8	1 427,4
06 - 07	49 336,2	6 152,3	2 054,8	5 018,6	7 418,6	3 519,7	1 883,2	2 202,8	3 185,2	2 339,6	4 712,9	1 432,3
07 - 08	50 822,7	6 498,7	2 339,8	5 271,2	7 445,5	3 562,3	1 949,8	2 239,0	3 211,9	2 368,9	4 774,3	1 449,0
08 - 09	53 345,3	6 642,9	2 503,5	5 968,8	8 131,7	3 601,3	2 033,8	2 229,3	3 256,4	2 440,4	5 668,4	1 482,5
09 - 10	54 718,1	6 795,6	2 495,9	6 094,7	8 379,3	3 668,0	2 089,2	2 240,4	3 259,3	2 498,1	5 701,9	1 497,9

Market Data: Price Formation

Each **observation** of Price (€ /MWh) and Quantity (MWh)

Volumes (MWh)

04-03-2018	Turnover at system price	NO1 Buy	NO1 Sell	NO2 Buy	NO2 Sell	NO3 Buy	NO3 Sell	NO4 Buy	NO4 Sell	NO5 Buy	NO5 Sell	SE1 Buy
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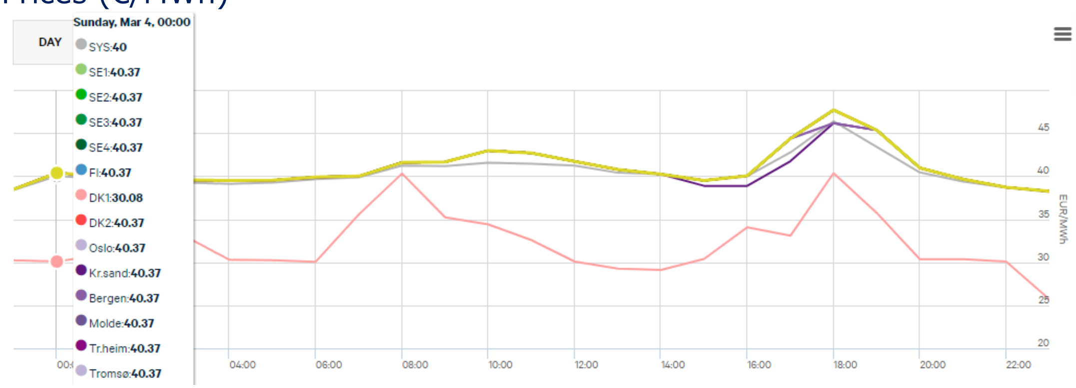
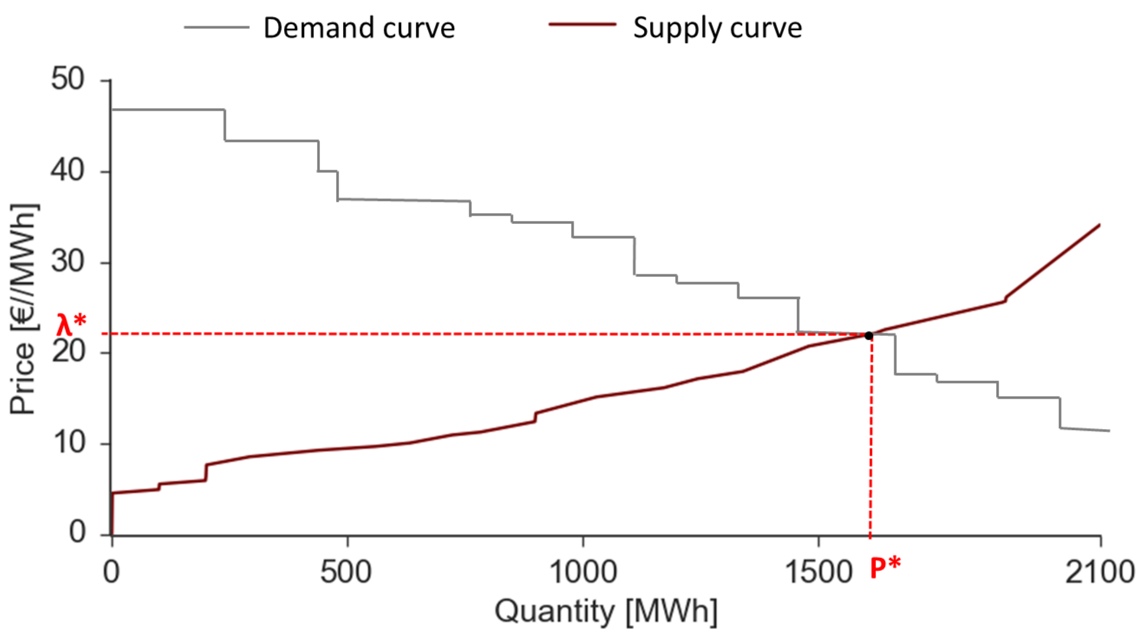
Prices (€/MWh)



Market Data: Price Formation

Each **observation** of Price (€/MWh) and Quantity (MWh)
is the **intersection point** of demand and supply curves at a given hour



Price Formation: Prior Knowledge

Inference Aim:
Use **observations** of maket data (price and quantity) to update our **prior knowledge** on the aggregate supply curve (**hidden variables**)

Question: how to model and update our prior knowledge based on these observations?

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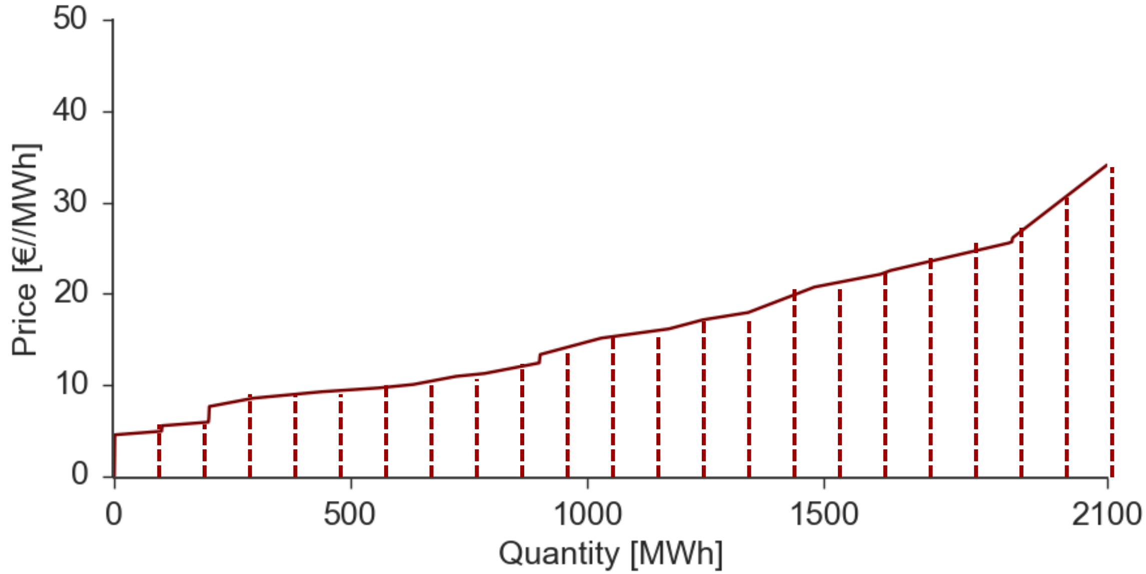

- **Parametrize** the aggregate supply curve for a specific hour (d,h)
- Arbitrary split into equal blocks (b)
- Assume piecewise linear supply function

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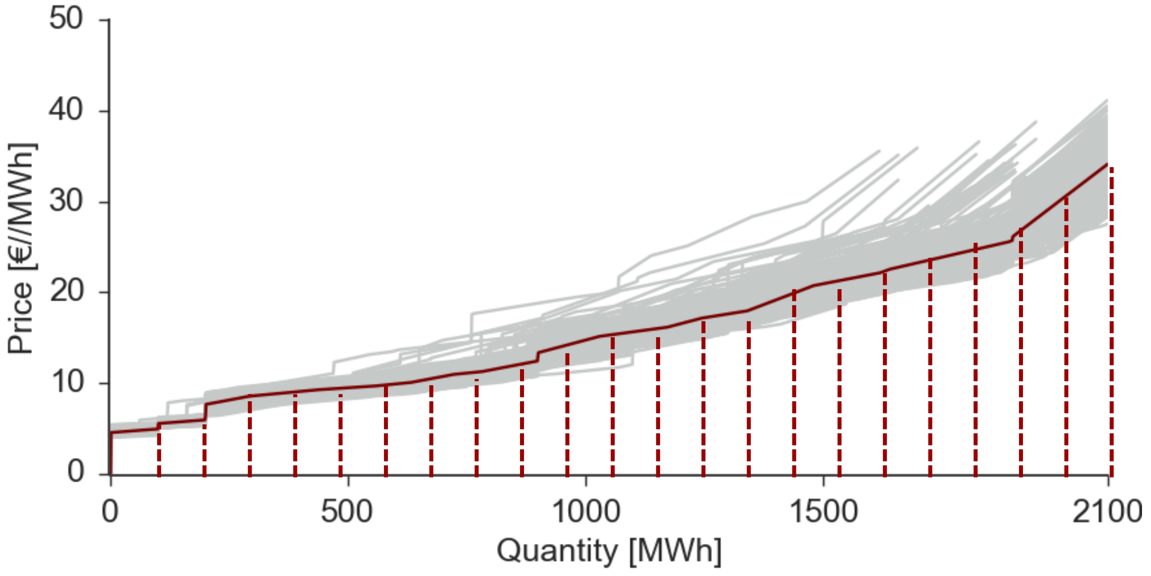
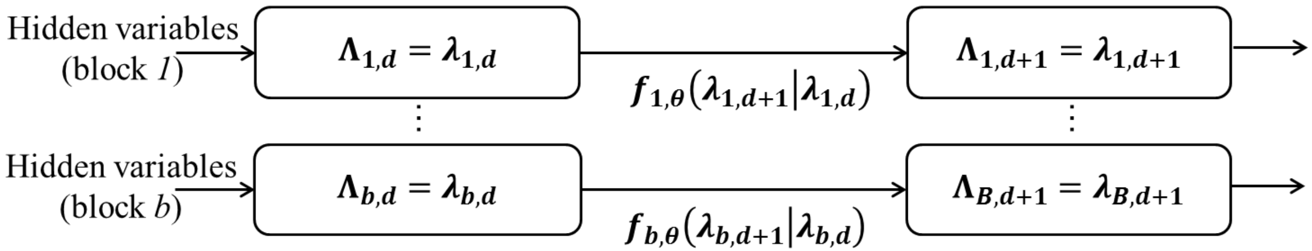


- Price parameters **updated** over time
- Price parameters unknown: model as **random variables**

Price Formation: Prior Knowledge



- Price parameters for each block modeled as **independent Markov chains**

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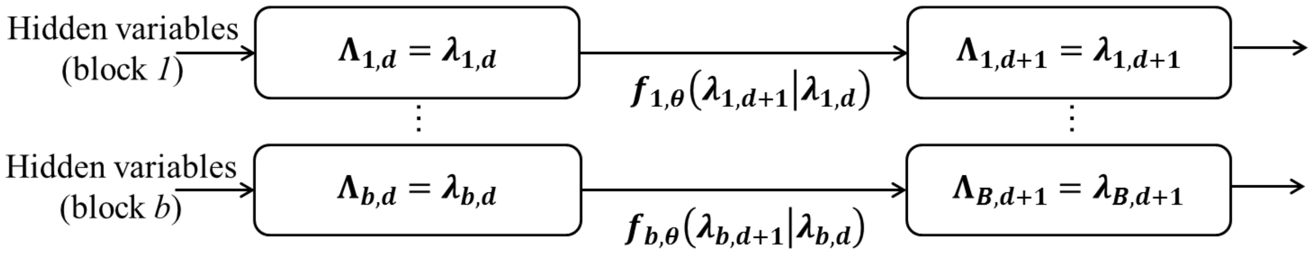
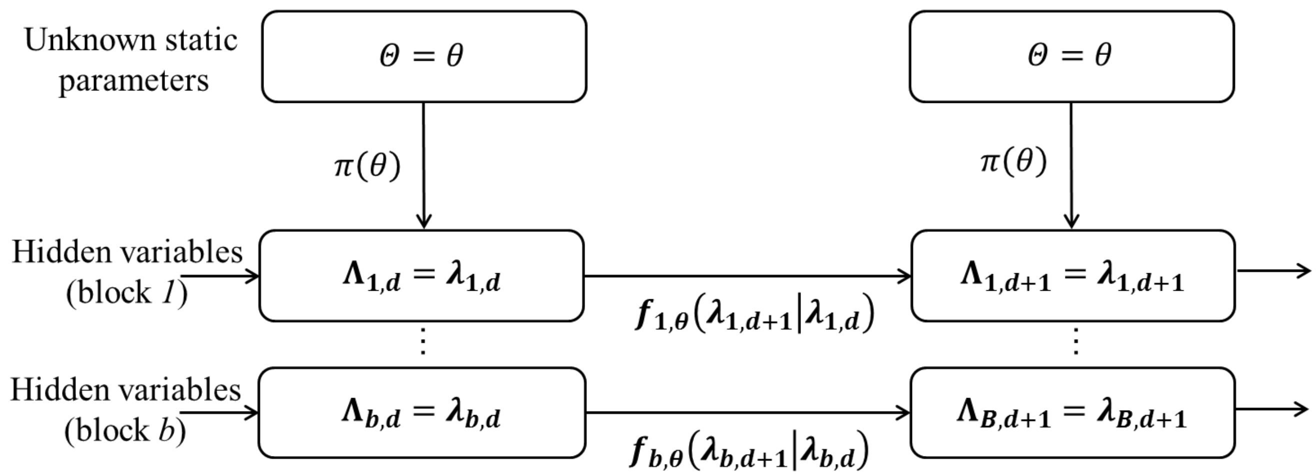


- Price parameters for each block modeled as **independent Markov chains**
- **Prior knowledge** on hidden variables: joint **transition function**

$$\boxed{\lambda_{d+1} \sim f(.\,|\lambda_d, \theta)}$$

Price Formation: Prior Knowledge



- Transition function also depends on **uncertain static parameters ($\boldsymbol{\theta}$)**

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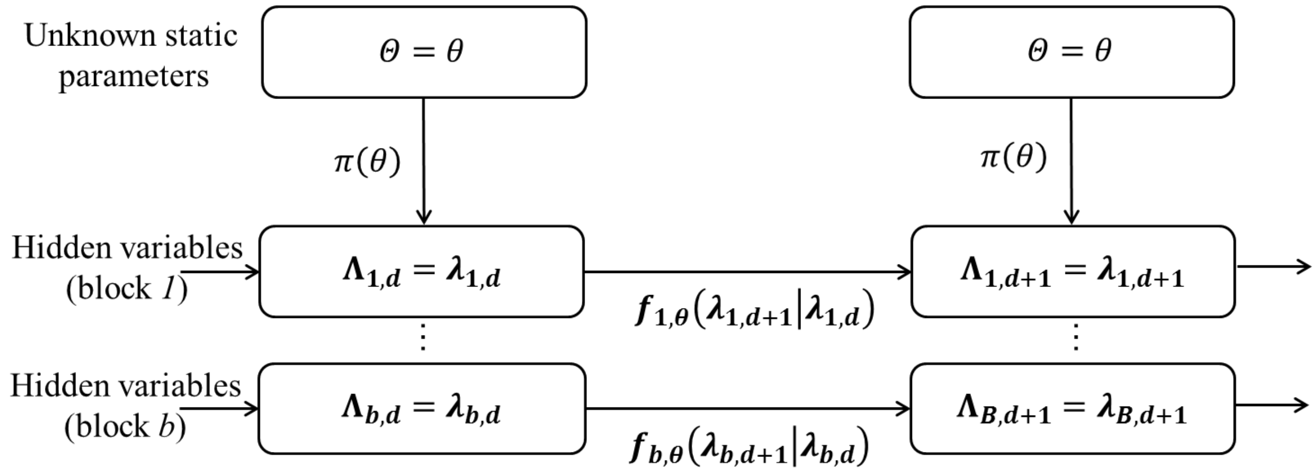
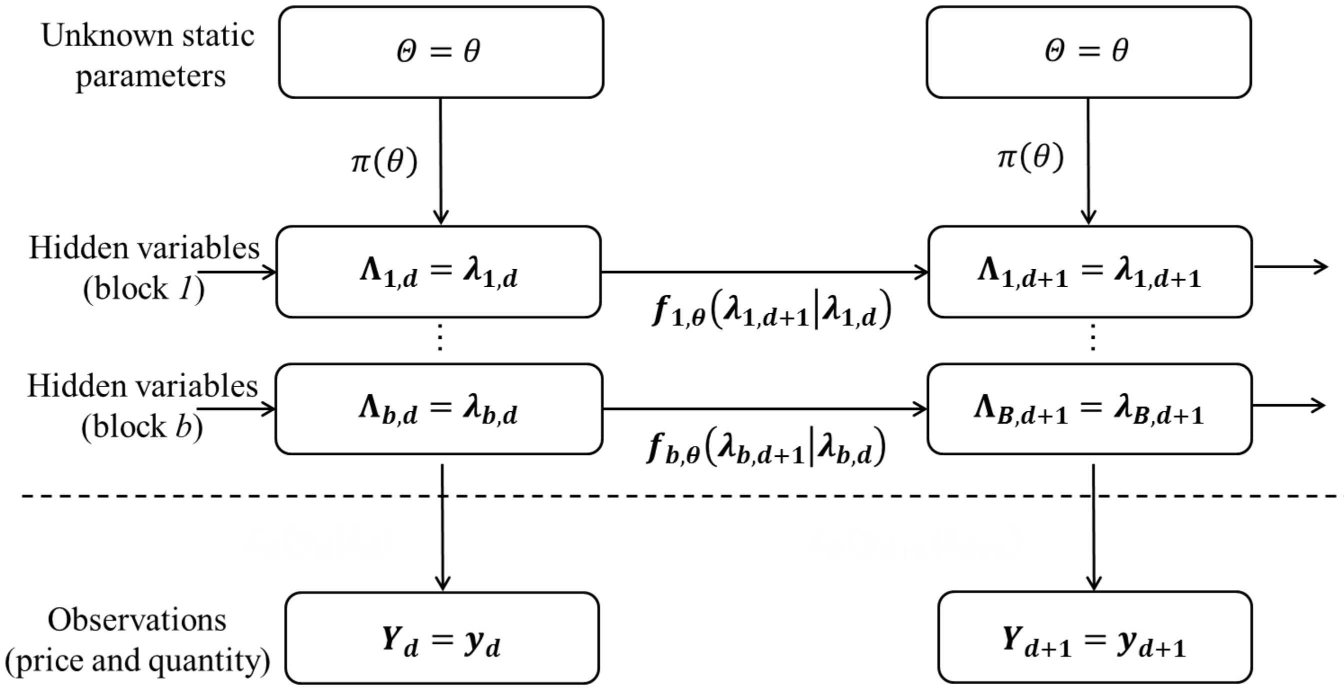


- Transition function also depends on **uncertain static parameters (θ)**
- **Prior knowledge** on hidden parameters: **prior density**

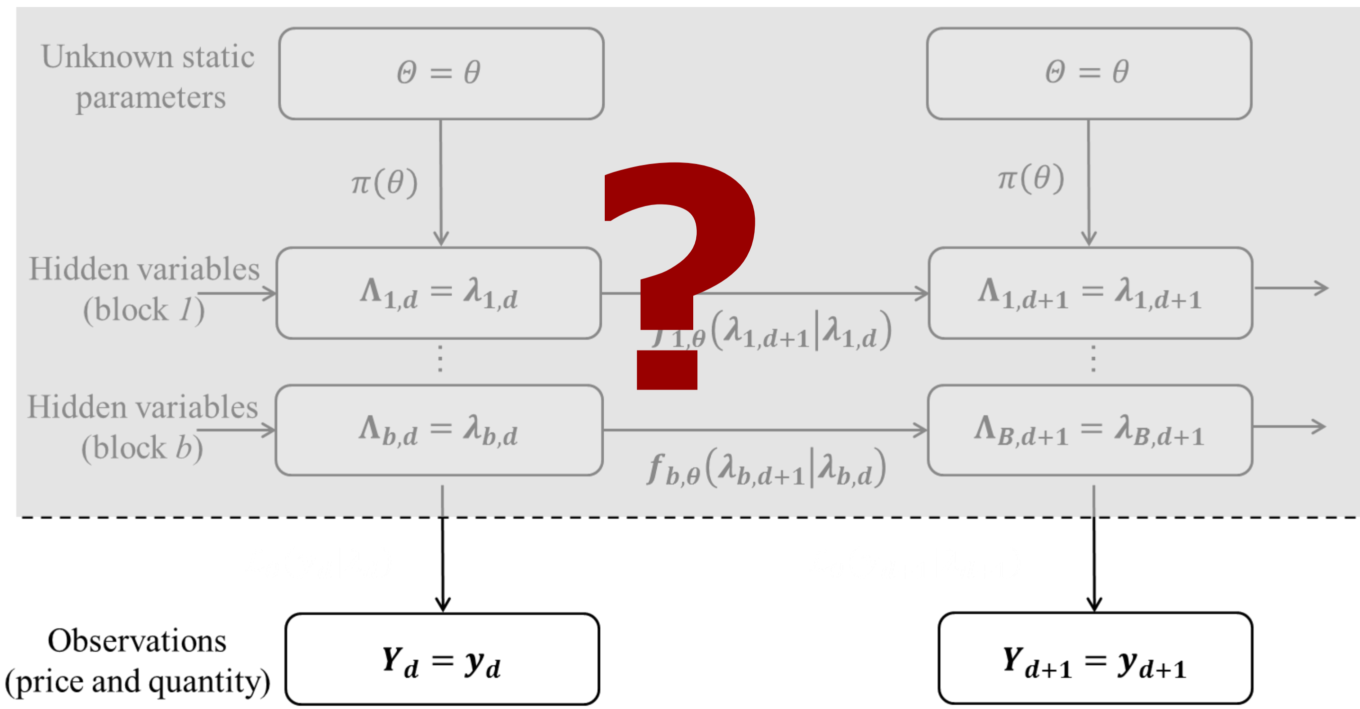
$$\boxed{\theta \sim \pi(.)}$$

Price Formation: Prior Knowledge



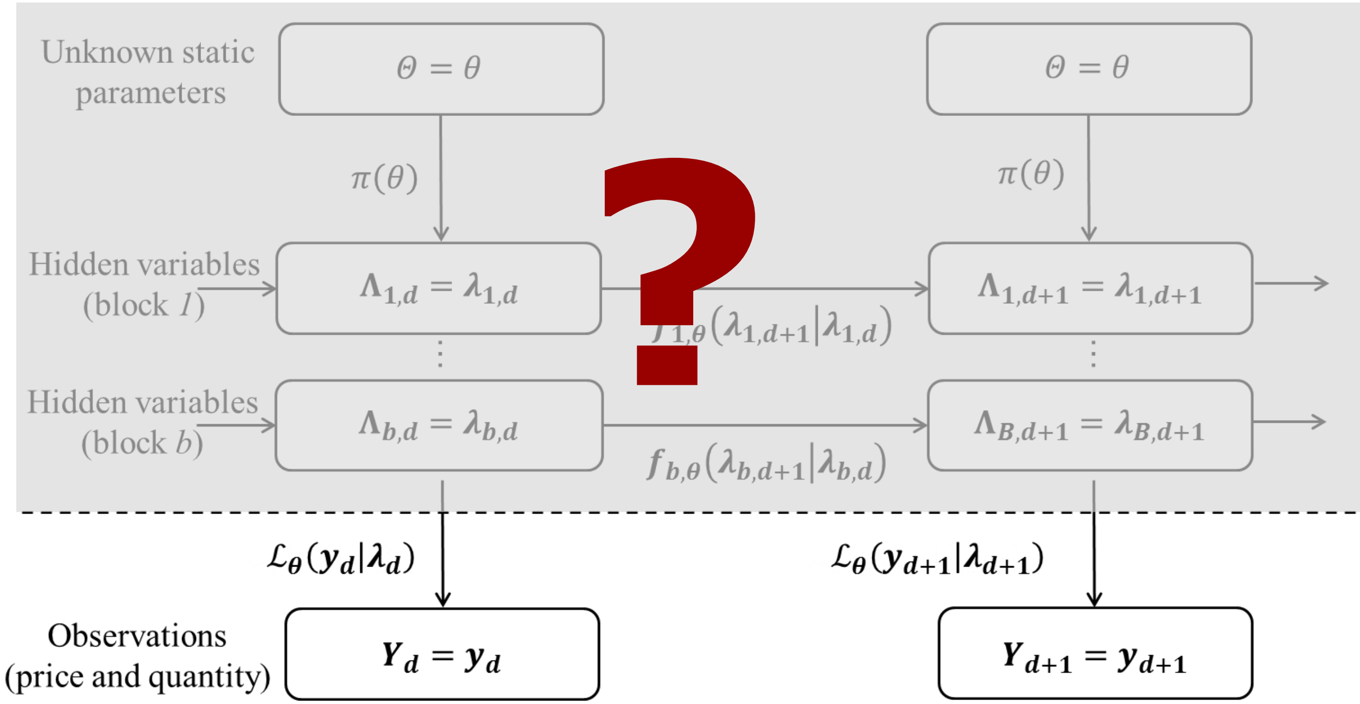
- Results of the market clearing: **Market data observations (y)**

Price Formation: Prior Knowledge



- Results of the market clearing: **Market data observations (y)**
- Hidden variables and parameters NOT observable

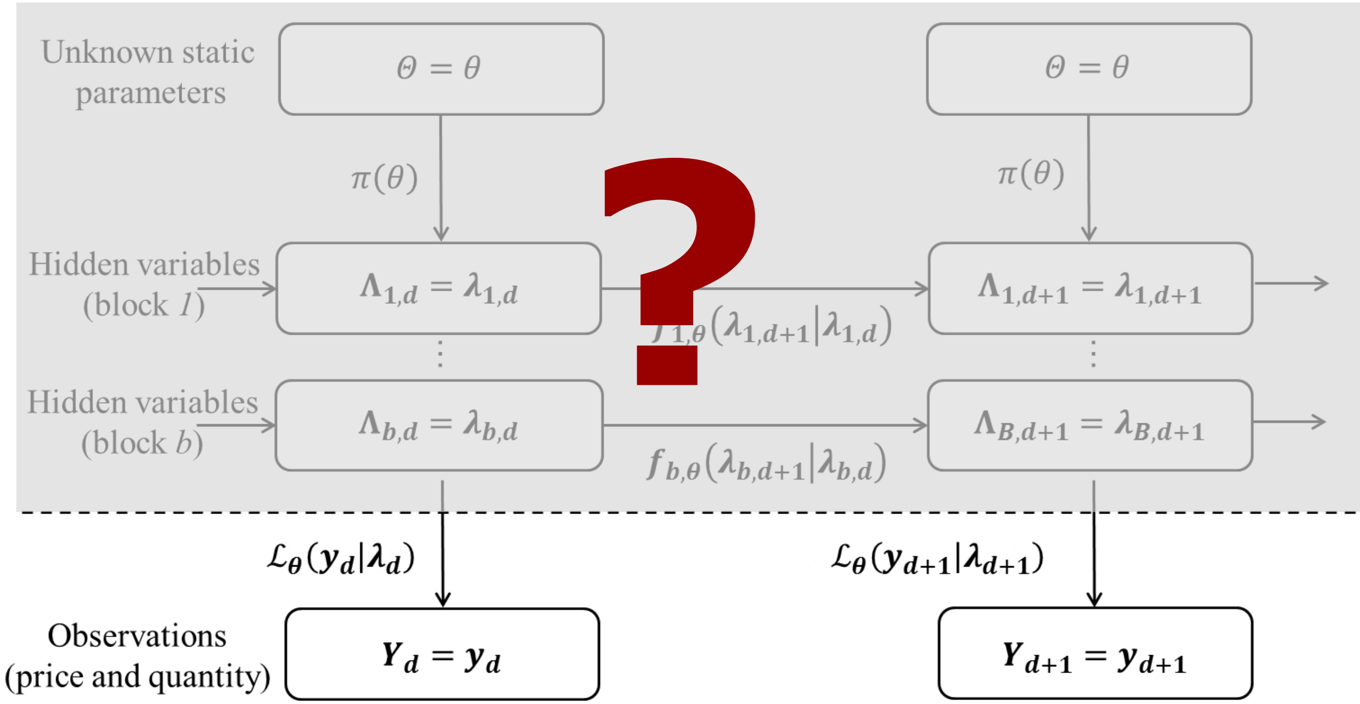
Price Formation: Prior Knowledge



- Observations depend on the hidden variables: **observations likelihood**

$$\boxed{y_d \sim \mathcal{L}(.\,|\lambda_d, \theta)}$$

Price Formation: Prior Knowledge



Question: how to update our prior knowledge based on these observations?

Bayesian Inference: Posterior Knowledge

- **Prosterior density** $P(\lambda_{1:D}, \theta | y_{1:D})$ represents posterior knowledge on hidden variables and parameters

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- **Bayes' Formula**:

$$\boxed{\Rightarrow\ P(A|B) \propto P(B|A) * P(A)}$$

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- **Bayes' Formula**:

$$\Rightarrow \boxed{P(\lambda_{1:D}, \theta | y_{1:D}) \propto \prod_{d} \mathcal{L}(y_d | \lambda_d, \theta) * f(\lambda_d | \lambda_{d-1}, \theta) * \pi(\theta)}$$

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Question: how to approximate this posterior density?

Markov Chain Monte Carlo (MCMC)

- **Iterative sampling** method
- Build a **Markov chain** $\{\lambda_{1:D}, \theta\}^{1:M}$ that converges towards the target density
- After a cerrtain number of steps, the sample $\{\lambda_{1:D}, \theta\}^{B:M}$ is approximately distributed acording to the target density

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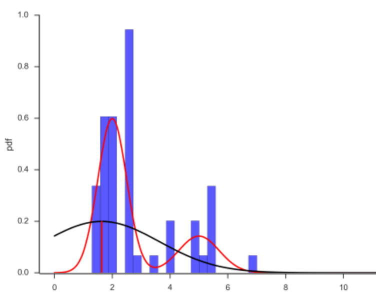
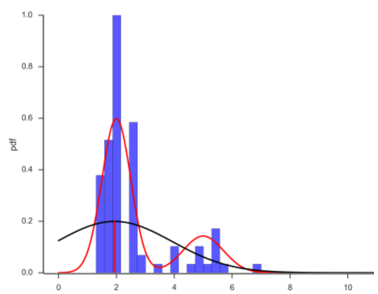
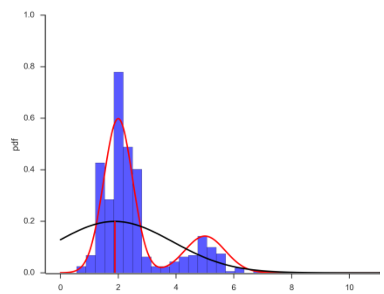
M=50



M=100



M=500



M=1000

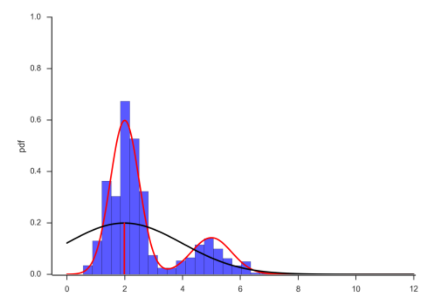
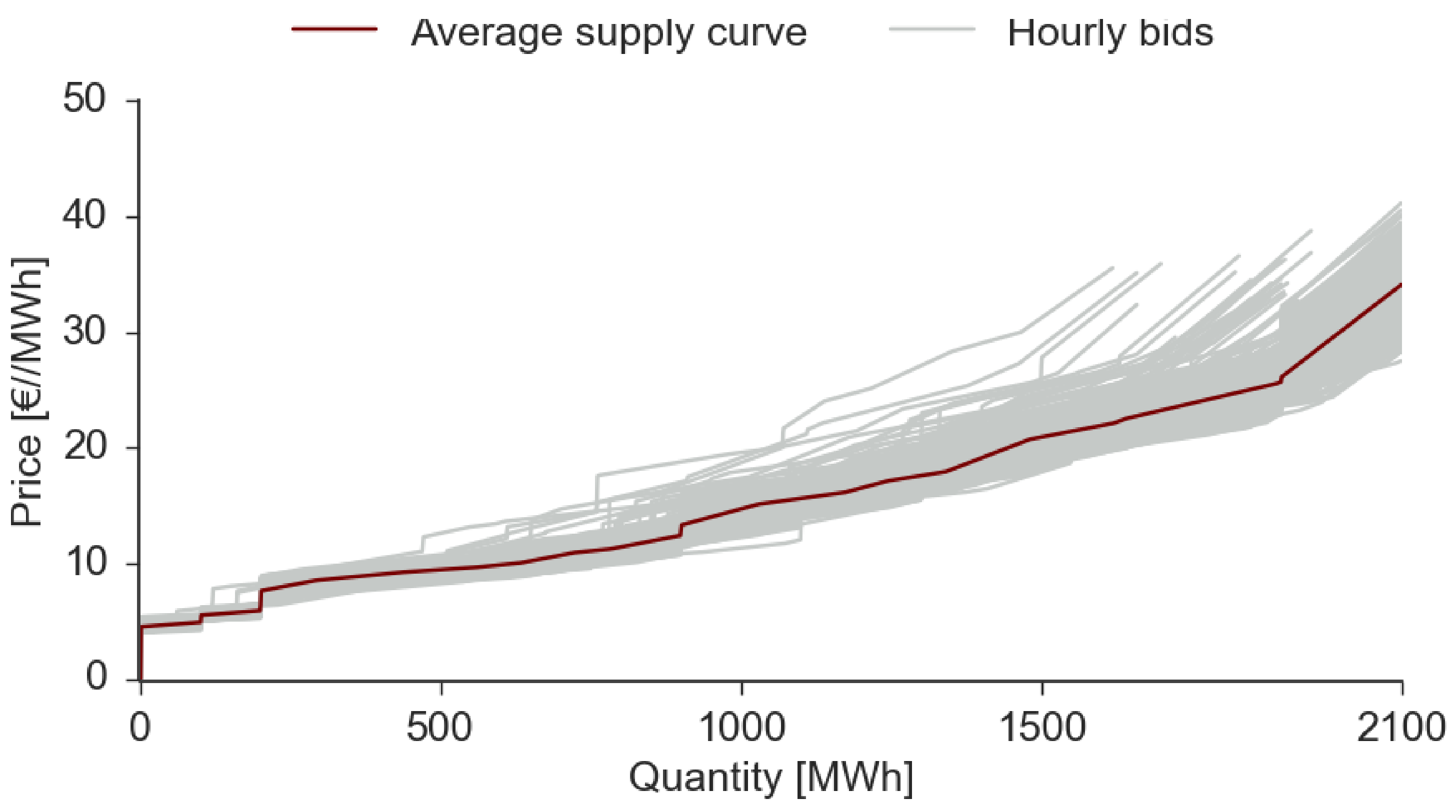


Figure: MCMC approximation of a density function

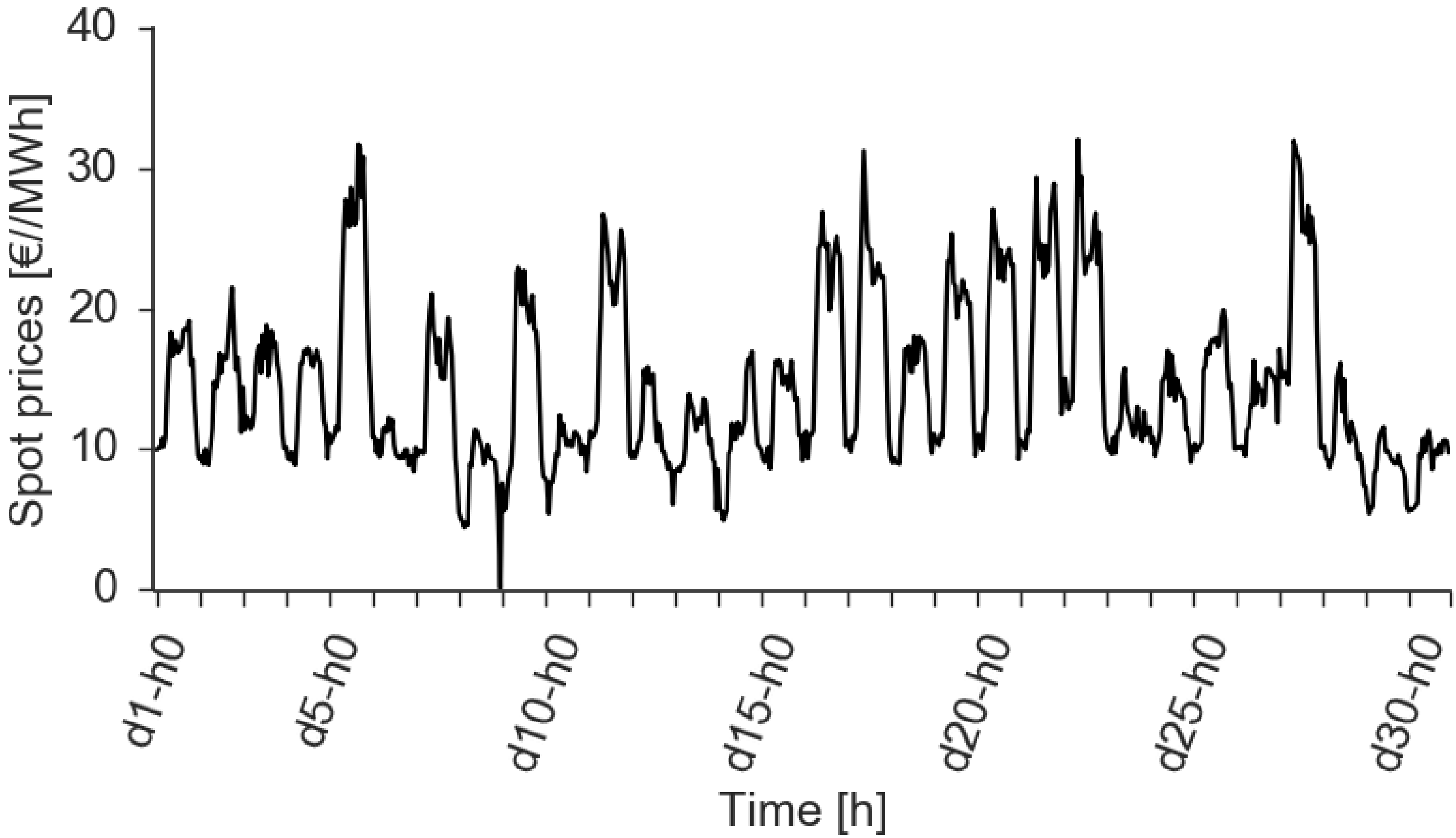
Case Study Setup

- Aggregate **Supply curves** over 1 month (unknown)
- **Simulate** a market clearing over 1 month



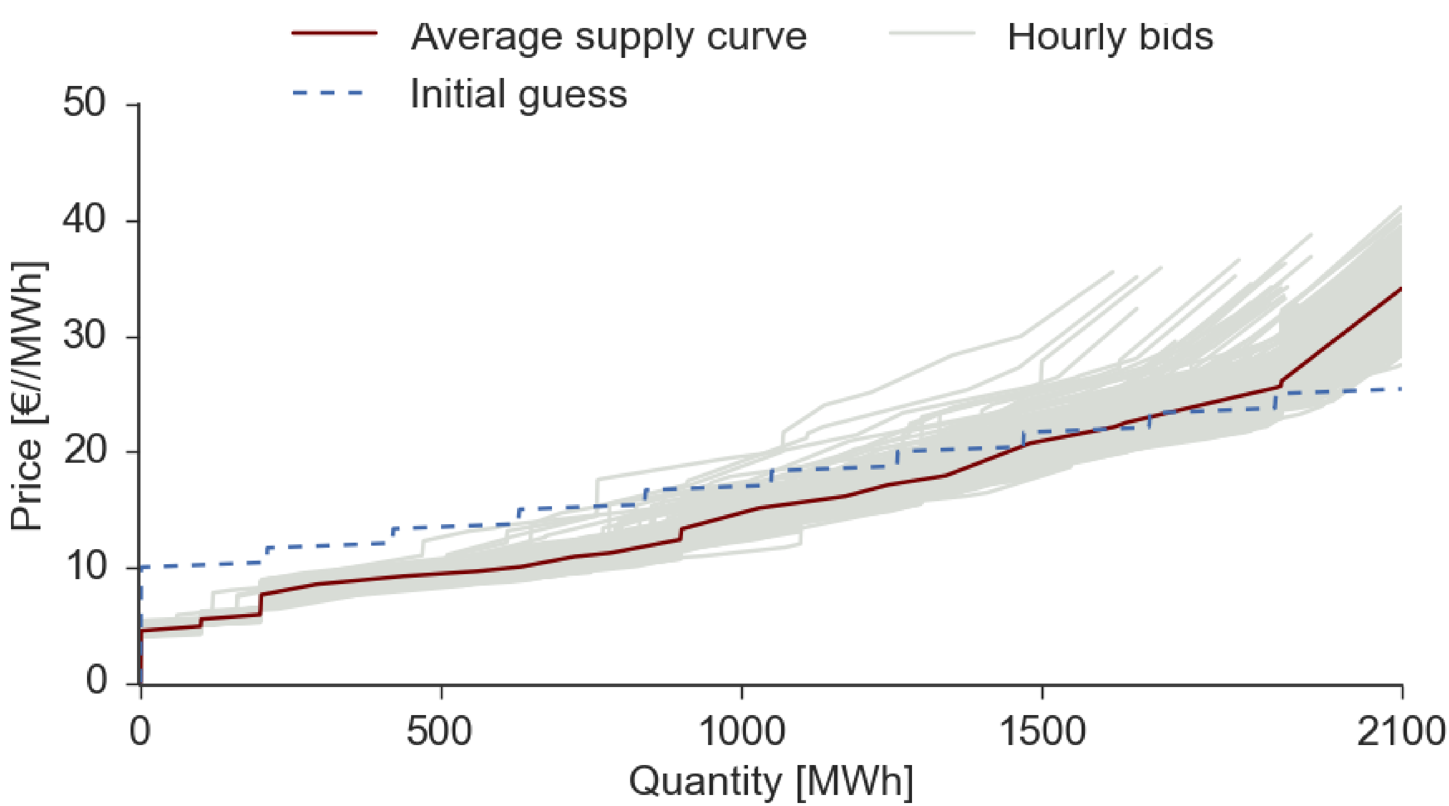
Case Study Setup

➢ **Observations**: Prices and quantities over 1 month

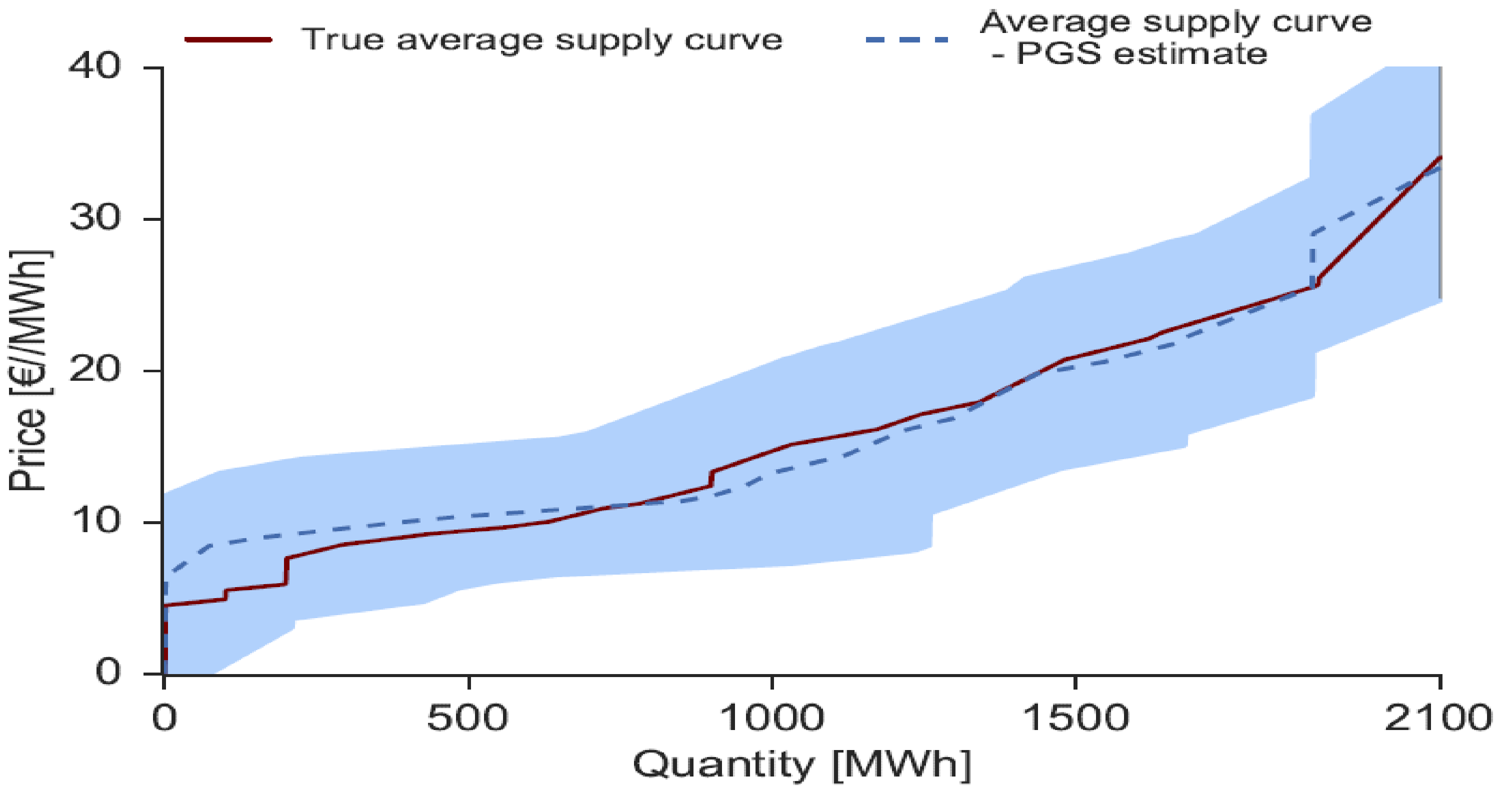


Case Study Setup

- **Prior Knowledge**: mean supply curve
- Assume independent Normal-Gamma **prior densities**



Case Study: Results



Conclusion and Discussions

- **Accurately estimate** the posterior density of the aggregate supply curve by iterative sampling
- Based on **market data observations**
- **Applications:** Input data for Strategic offering

Challenges:

- Design an efficient algorithm in high dimension
- Computational complexity: scalable for large scale systems?
- Integrate unit commitment data, transmission constraints in the model

Reference
Mitridati, L., & Pinson, P. (2017). A Bayesian Inference Approach to Unveil Supply Curves in Electricity Markets. *IEEE Transactions on Power Systems*.

Thank you for your attention !!

<u>Contact</u>: Lesia Mitridati (lemitri@elektro.dtu.dk)