

# Stochastic Defense Against Complex Grid Attacks

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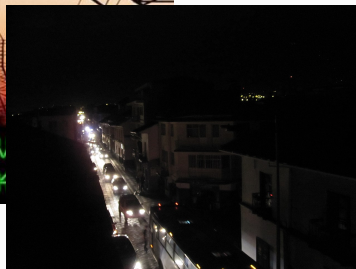
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# AC Power Flow Problem

(line physics): admittance matrix

$$Y_{km} = \begin{bmatrix} y_{kk} & y_{km} \\ y_{mk} & y_{mm} \end{bmatrix} \in \mathbb{C}^{2 \times 2}$$

$$V_k = |V_k|e^{j\theta_k}$$

$$V_m = |V_m|e^{j\theta_m}$$



$$j = \sqrt{-1}$$

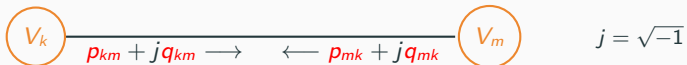
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Active (real) and reactive (imaginary) **power flows**:

$$p_{km} = y_{kk}^{re} |V_k|^2 + y_{km}^{re} |V_k| |V_m| \cos(\theta_k - \theta_m) + y_{km}^{im} |V_k| |V_m| \sin(\theta_k - \theta_m)$$

$$q_{km} = -y_{kk}^{im} |V_k|^2 - y_{km}^{im} |V_k| |V_m| \cos(\theta_k - \theta_m) + y_{km}^{re} |V_k| |V_m| \sin(\theta_k - \theta_m)$$

(where  $x = x^{re} + jx^{im}$ )

# Optimal Power Flow Problem

Find a solution to:

- minimize  $c(\{P_k^g\}_k)$  *(usually a quadratic function)*
- for each bus  $k$  *(power-injection balance)*

$$\sum_{km \in \delta(k)} (p_{km} + j q_{km}) = (P_k^g + j Q_k^g) - (P_k^d + j Q_k^d)$$

- for each branch  $km$

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**Non-convex quadratic problem!**

⇒ Solvers: IPOPT, others. Matpower package for Matlab



# “Cyber-Physical” attacks

Fact or fiction?

- An adversary carries out a physical alteration of a grid (example: disconnecting a power line)
- This is coordinated with a modification of sensor (PMU) signals – a **hack**
- The goal is to disguise, or keep completely hidden, the nature of the attack and its likely consequences

- All, or mostly, DC-based
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- Intelligent procedures for enriching state estimation so as to detect and reconstruct attacks
- Liu, Ning Rieter (2009), Kim and Poor (2001)
- Deka, Baldick, Vishwanath (2015)
- Soltan, Yannakakis, Zussman (2015 - )

- Attacker disconnects lines plus alters sensor output in an (unknown) zone of the grid
- As a result, the equation

$$B\theta = p^g - p^d$$

is wrong because  $B$  is incorrect and measurements  $\theta$  are (sparsely) false

- A statistical procedure to try to “fit” a correction to  $B$  and a discovery of false data

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  - **Cause line overloads that remain hidden**
- We assume **full PMU deployment**. Everything is **AC** based.

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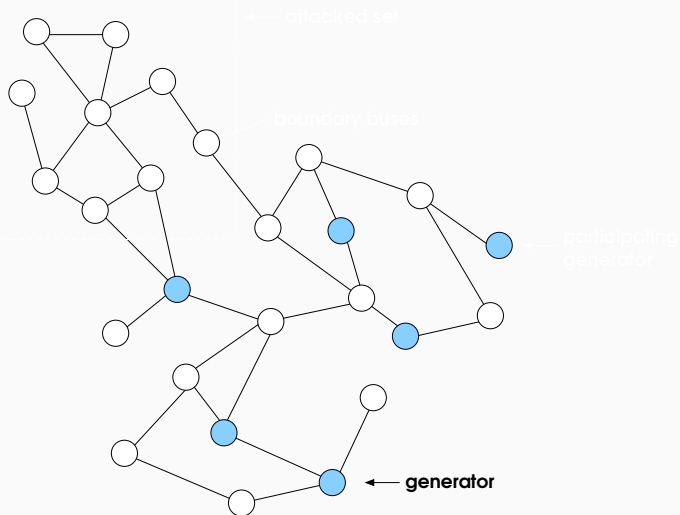
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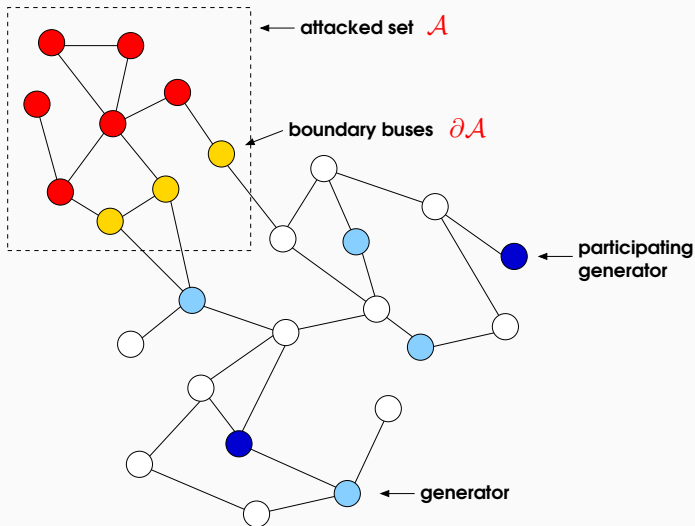
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- Post-attack: attacker cannot recompute much and only relies on adding “noise” to computed distorted signals

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- Two power flow solutions; each must satisfy AC power flow
- A generation change consistent with AGC (automatic generation control)

# Undetectable attack: formulation (abridged!)

$$\text{Max } (p_{uv}^T)^2 + (q_{uv}^T)^2 \quad (1a)$$

s.t.

$$\forall k \in \mathcal{A}^C \cup \partial\mathcal{A}, \quad |V_k^T| = |V_k^R|, \quad \theta_k^T = \theta_k^R \quad (1b)$$

$$\forall k \in \mathcal{A}: \quad -(P_k^{d,R} + jQ_k^{d,R}) = \sum_{km \in \delta(k)} (p_{km}^R + jq_{km}^R), \quad P_k^{d,R} \geq 0 \quad (1c)$$

$$-(P_k^{d,T} + jQ_k^{d,T}) = \sum_{km \in \delta(k)} (p_{km}^T + jq_{km}^T), \quad P_k^{d,T} \geq 0 \quad (1d)$$

$$\forall k \in \mathcal{A}^C \setminus \mathcal{R}, \quad \hat{P}_k^g - \hat{P}_k^d + j(\hat{Q}_k^g - \hat{Q}_k^d) = \sum_{km \in \delta(k)} (p_{km}^T + jq_{km}^T) \quad (1e)$$

$$\forall k \in \mathcal{R}: \quad P_k^g - \hat{P}_k^d + j(Q_k^g - \hat{Q}_k^d) = \sum_{km \in \delta(k)} (p_{km}^T + jq_{km}^T) \quad (1f)$$

$$P_k^g - \hat{P}_k^g = \alpha_k \Delta \quad (1g)$$

operational limits on all buses, generators, and

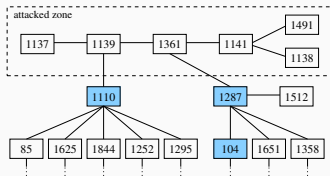
lines (outside of attacked area) (1h)

all  $p_{km}^T, q_{km}^T$  related to  $V_k^T, V_m^T$  and

all  $p_{km}^R, q_{km}^R$  related to  $V_k^R, V_m^R$  through AC power flow laws (1i)

# A large-scale example

From case2746wp (that has 2746 buses) from the Matpower case library



bus $k$	bus $m$	$p_{km}^T$ $p_{km}^R$	$q_{km}^T$ $q_{km}^R$	$\ (p_{km}^T, q_{km}^T)\ $ $\ (p_{km}^R, q_{km}^R)\ $	$S_{km}^{max}$
1139	1137	3.36 3.36	2.66 2.66	4.29 4.28	114.00
1361	1141	229.01 108.51	10.49 10.49	<b>229.25</b> 109.02	114.00
1141	1491	13.46 6.20	2.41 2.39	13.68 6.64	114.00
1141	1138	209.25 98.06	4.44 5.24	<b>209.29</b> 98.20	114.00

Undetectable attack with strong overloads on branches (1361, 1141) and (1141, 1138)

# Ideal attack: follow-up

Following the attack, attacker needs to **report dynamic data** that satisfy:

- current-voltage consistency:  $I_{km}^R(t) \approx y_{kk} V_k^R(t) + y_{km} V_m^R(t)$
- power-injection consistency:  $\sum_{km \in \delta(k)} V_k^R(t) I_{km}^R(t)^* \approx \text{net-injection at } k$

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We assume that the attack is perpetrated in ambient conditions, and consider two scenarios:

**1. Noisy Data Attack.** For any bus and line in  $\mathcal{A}$  the attacker reports

$$V_k^R(t) = V_k^R + \nu_k(t), \quad I_{km}^R(t) = I_{km}^R + \mu_{km}(t)$$

where  $\nu_k(t)$  and  $\mu_{km}(t)$  are drawn from a small variance, zero mean distribution.

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**2. Data Replay Attack.** Attacker supplies previously observed/computed series  $V_k^R(t)$ ,  $I_{km}^R(t)$ .



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**Random Defense Strategy.** Iterate the following steps:

1. For each generator  $k \in \mathcal{G}$ , randomly choose  $\delta_k$  such that  $\sum_{k \in \mathcal{G}} \delta_k \approx 0$
2. Command each generator to change its output to  $P_k^g + \delta_k$
3. Identify inconsistencies in the observed PMUs

Remark: Feasibility in step 1, OPF-like problem

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Sensed values  $V_k^S, V_m^S, I_{km}^S, I_{mk}^S$  must satisfy following **Criteria**:

1.  $|V_k^S - y_{mk}^{-1}(I_{mk}^S - y_{mm}V_m^S)| < \frac{2\tau|y_{mk}^{-1}|}{1-\tau}(|I_{mk}^S| + |y_{mm}||V_m^S|)$
2.  $|I_{km}^S - y_{kk}V_k^S - y_{km}V_m^S| < \frac{\tau}{1-\tau}(|I_{km}^S| + |y_{kk}||V_k^S| + |y_{km}||V_m^S|)$

If reported phasors do not satisfy these criteria, then line  $km$  is **flagged**

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**Lemma.** *Suppose that*

$$|V_k^T(*) - V_k^R(0)| > \frac{2\tau|y_{km}^{-1}|}{1-\tau} (|I_{mk}^T(*)| + |y_{mm}||V_m^T(*)|) + \frac{2\tau|y_{ka}^{-1}|}{1-\tau} (|I_{ak}^R(0)| + |y_{aa}||V_a^R(0)|)$$

*Then, it is impossible for the noise data attacker to statistically satisfy Criterion 1 on both lines  $ak$  and  $mk$*

Pf. sketch: Use Criterion 1 for lines  $ak$  and  $mk$ .

# Defense: Identifying Inconsistencies, Experiment

	Experiment 1	Experiment 2
$\sum_{k \in \mathcal{G}}  \delta_k $	463.48	1220.81

Line ( $k = 1139, a = 1137$ )

$ V_a^R(0)  \angle \theta_a^R(0)$	$1.0919 \angle -6.993^\circ$	$1.0919 \angle -6.993^\circ$
$I_{ak}^R(0)$	$-0.0275 + 0.0281j$	$-0.0275 + 0.0281j$

Line ( $k = 1139, m = 1110$ )

$ V_m^T(*)  \angle \theta_m^T(*)$	$1.0309 \angle -7.822^\circ$	$1.0391 \angle -7.848^\circ$
$I_{mk}^T(*)$	$0.0905 - 0.4976j$	$0.1289 - 0.4901j$

Voltages at  $k = 1139$

$ V_k^R(0)  \angle \theta_k^R(0)$	$1.0919 \angle -6.991^\circ$	$1.0919 \angle -6.991^\circ$
$ V_k^T(*)  \angle \theta_k^T(*)$	$1.0104 \angle -7.822^\circ$	$1.0187 \angle -7.936^\circ$

Lemma applied to bus  $k = 1139$

Ratio	1.913	1.732
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Consider the vector of post-attack voltage angles  $\theta^R(t) = (\theta_k^R(t) : k \in \mathcal{N})$ .

Control center **can learn statistics** of  $\theta^R$ , denote by  $\Omega$  its covariance matrix.

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Consider:

- $\lambda_1 \geq \dots \geq \lambda_r > 0$  eigenvalues of  $\Omega$  larger than certain  $\epsilon > 0$
- $w_1, \dots, w_r$  its corresponding eigenvectors
- $\Gamma > 0$  larger compared to  $\epsilon$
- a zero-mean distribution  $\mathcal{P}$  with support in  $[-1, 1]$
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## Covariance Defense Procedure. Iterate:

V1. Choose a nonzero vector  $v \in \mathbb{R}^n$  such that

(a)  $(Bv)_k = 0$  for all  $k \notin \mathcal{F}$

(b)  $w_i^\top v = 0$  for  $i = 1, \dots, r$

(c) for each  $k \in \mathcal{F}$ ,  $P_k^g \pm \Gamma(Bv)_k$  is feasible for generator  $k$

V2. For  $s = 1, 2, \dots$  perform epoch  $s$ :

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If  $\delta = \mathbf{x}\Gamma\mathbf{v}$ , then  $\mathbf{E}[\delta] = 0$ ,  $\mathbf{Var}(\delta) = \mathbf{Var}(\mathbf{x})\Gamma^2\mathbf{v}\mathbf{v}^\top$

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**Lemma:** Suppose  $\mathbf{x}$  is stochastically independent of ambient noise.

Then, under DC model,  $\mathbf{Var}(\hat{\theta}^\top) = \mathbf{Var}(\theta^\top) + \mathbf{Var}(\mathbf{x})\Gamma^2\mathbf{v}\mathbf{v}^\top$ .



- “Ideal” attacks that cause and hide overloads are feasible on large networks
- Two realistic mechanisms to detect an attack, when suspected, changing the generation at certain buses
  - Identifying the boundary lines of the attacked zone, or
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